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MAXIMIZING INFRARED EXTINCTION COEFFICIENTS FOR METAL DISCS, RODS, AND SPHERES

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RESEARCH AND TECHNOLOGY DIRECTORATE

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Infrared spectral extinction coefficients, extinction cross sections per volume of material, are computed for metal disks, rods, and spheres as a function of dimensions, conductivity, and wavelength. Computations for rods use a combination of infinite cylinder and Rayleigh finite and perfectly conducting prolate spheroid scattering theories. Disk computations use a combination of diffraction, thin film optics, Rayleigh finite, and perfectly conducting oblate spheroid scattering theories. Sphere computations use a combination of Geometric Optics, Anomalous Diffraction, and Rayleigh scattering theories. Complex conductivities are predicted as a function of wavelength using the Drude model along with thin wire and thin film models for boundary limited electron mean free path. Spectral extinction coefficients are maximized using surface plots to identify optimal ranges for particle dimensions and conductivities at a particular wavelength.

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PREFACE

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MAXIMIZING INFRARED EXTINCTION COEFFICIENTS
FOR METAL DISCS, RODS, AND SPHERES

1. INTRODUCTION

Infrared spectral extinction coefficients, extinction cross sections per volume of material, are computed for metal disks, rods and spheres as a function of dimensions, conductivity and wavelength. Computations for rods use a combination of infinite cylinder and Rayleigh finite and perfectly conducting prolate spheroid scattering theories. Disk computations use a combination of diffraction, thin film optics, Rayleigh finite and perfectly conducting oblate spheroid scattering theories. Sphere computations use a combination of Geometric Optics, Anomalous Diffraction, and Rayleigh finite and perfect conducting sphere scattering theories. Complex conductivities are predicted as a function of wavelength using the Drude model along with thin wire and thin film models for boundary limited electron mean free path. Spectral extinction coefficients are maximized using extinction coefficient surface plots in parameter hyperspace to identify optimal ranges for particle dimensions and conductivities at a particular wavelength. Dramatic differences are evident in the way that essentially one, two and three dimensional metal particles interact with infrared radiation.

2. SMOKE MECHANISMS FOR DEFEAT OF SENSORS

2.1 Effective Smoke Screening: Active vs Passive and Infrared vs Visible Systems

There are differences between smoke screening against an active system compared to smoke screening against a passive system in the infrared. Smoke reduces a passive system signal (target radiance) by one-way attenuation and reduces an active system signal by two-way attenuation. Smoke clouds superimpose scattered ambient radiance and cloud emitted radiance onto both the attenuated signal and surrounding attenuated background for active and passive systems. This changes the noise level, which is proportional to the square root of signal and background radiance for ideal photon noise limited detection. For active systems the scattered probe radiance is removed by range gating and scattered ambient radiance and cloud emitted radiance become a smaller fraction of the attenuated signal and background as probe radiance increases. Unlike the visible spectral region where ambient radiance has a diffuse sky component representing as little as 10% of the incident direct solar radiance, ambient infrared radiance is generally more isotropic especially under rainy, humid or cloudy conditions and therefore less influenced by differential scatter asymmetry within the smoke. Also unlike the visible spectral region where increasing smoke single scatter albedo always increases the smoke radiance superimposed onto the attenuated signal and background, in the infrared the combined effects of greater scatter and less absorption will result in less cloud radiance when the drop in thermal emission outweighs the increase in scattered ambient radiance. This occurs when smoke cloud temperature exceeds ambient temperatures.

2.2 Smoke Reductions in Signal to Noise Ratio

The goal of effective smoke screening is to reduce signal to noise ratio using the least amount of smoke material. In the infrared region trade-offs associated with increasing noise by adjusting combinations of cloud emittance, reflectance and scattered light transmittance versus reducing signal by reducing cloud beam transmittance is discussed in a separate report (Embury, in preparation). In the visible spectral region trade-offs associated with increasing noise by

increasing cloud reflectance and scattered light transmittance is discussed (Embury, in preparation) under a variety of solar illumination conditions versus reducing signal by reducing cloud beam transmittance. Those reports discuss how cloud thermal emission increases and cloud reflectance and scattered light transmittance decrease as smoke particle single scatter albedo decreases and the signal to noise ratio tradeoffs associated with changing single scatter albedo, extinction coefficient, differential scattering pattern, cloud temperature and ambient illumination.

Minimizing signal by reducing cloud beam transmittance using a minimal amount of smoke material is discussed here. Signal beam transmittance is treated by breaking down the optical depth αCL into two parts. The first part is the extinction coefficient α, the extinction (scatter plus absorption) cross section per volume m^2/cc or mass m^2/g of aerosol material. The second part is the concentration path length product CL, the volume or mass of aerosol per cloud area in the image plane, where C is the aerosol volume cc/m^3 or mass g/m^3 concentration and L is the pathlength in meters through the cloud. Maximizing the extinction coefficient minimizes the quantity of material CL required to reach a specified optical depth.

2.3 Extinction Coefficients

Volume extinction coefficients depend on optical efficiency factor Q, geometric cross section G and particle volume V as indicated by the relationship

$$\alpha = \mathrm{QG/V}.$$

For convex particles the geometric cross section averaged over random orientation is one fourth the surface area S so that[1]

$$\alpha = \mathrm{QS/4V}.$$

Therefore, the volume extinction coefficient can be increased by either increasing Q or by increasing the surface area to volume ratio. In order to significantly increase Q over a narrow wavelength range we can choose a combination of particle properties that produce a resonance. This requires particles that are on the order of the wavelength in size and that fairly monodisperse in size and shape. Increasing the surface area to volume ratio means reducing particle size but eventually the particles become small enough that increases in surface to volume ratio can be offset by decreases in Q. Spherical particles small compared to the wavelength have a resonance at $m = (0, \sqrt{2})$ which broadens to a wide range of metallic refractive indices for rod (fiber) and disk (flake) shapes[2]. This resonance maintains Q at levels around 2 in the case of metal discs and much higher levels in the case of rods. This resonance is not significantly decreased by size distribution.

Volume extinction coefficients are related to mass extinction coefficients using particle density ρ

$$\alpha\left[\mathrm{m^2/g}\right] = \alpha\left[\mathrm{m^2/cc}\right] / \rho\left[\mathrm{g/cc}\right]$$

and it is convenient to express the surface area to volume ratio in micrometers μ to obtain the extinction coefficient directly in m^2/cc. It should be mentioned at this point that there are three ways to increase the mass extinction coefficient; the same two ways that were just mentioned to increase the volume extinction coefficient plus reducing the particle density as in the case of hollow or foam particles, although the benefits of this third method will eventually be offset by decreases in Q at very low densities.

The remainder of this report involves computation of the volume extinction coefficients α and Q which are functions of particle size, shape, wavelength and complex refractive index which in turn depends on constitutive properties as well as wavelength and minimum dimension in the case of nanoparticles.

3. CALCULATIONS OF EXTINCTION COEFFICIENTS

3.1 Maximizing Extinction Coefficients of Polydispersions of Spheres

Scattering by polydispersions of spheres predicted by the Mie solution can be used to represent scattering by collections of non-spherical particles as long as they are roughly isometric (equal dimensions along any three orthogonal axes) and have size, shape and orientation distributions broad enough to damp out resonances. The advantage of using the sphere solution is that it requires less computational time than popular non-spherical techniques like the Extended Boundary Condition Method, the Discrete Dipole Approximation and the Separation of Variables Method for a spheroid[3].

The Mie solution for the scattering cross section of a sphere is[1, 2]

$$C_{sca} = \frac{W_{sca}}{I_i} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2 + |b_n|^2\right)$$

and the Mie solution for the extinction cross section of a sphere is

$$C_{ext} = \frac{W_{ext}}{I_i} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}\left(|a_n|^2 + |b_n|^2\right)$$

where $k = 2\pi/\lambda$ and for a sphere with complex refractive index m and diameter D having a size parameter $x = \pi D/\lambda$,

$$a_n = \frac{m\psi_n(mx)\psi_n'(x) - \psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x) - \xi_n(x)\psi_n'(mx)}$$

$$b_n = \frac{\psi_n(mx)\psi_n'(x) - m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)}.$$

The Riccati-Bessel functions

$$\psi_n(\rho) = \rho j_n(\rho), \qquad \xi_n(\rho) = \rho h_n^{(1)}(\rho)$$

may be written in terms of the Spherical Bessel functions

$$j_n(\rho) = \sqrt{\frac{\pi}{2\rho}}J_{n+1/2}(\rho)$$

$$y_n(\rho) = \sqrt{\frac{\pi}{2\rho}} Y_{n+1/2}(\rho)$$

which are defined in terms of the Bessel functions of the first kind J and of the second kind Y.

Spherical aerosol size distributions average out extinction coefficient resonances in Q yielding a simple result for optimum diameter and complex refractive index producing maximum achievable extinction coefficients over a wavelength band. Mie calculations show that the maximum achievable extinction coefficient for a spherical polydispersion over a wavelength band is approximately

$$\alpha_{max}[\mathrm{m^2/cc}] \approx 3/\mathrm{D_{opt}}[\mu],$$

D_{opt} is the minimum diameter maintaining $Q \geq 2$ over the wavelength band. Mie calculations also indicate that the optimum diameter as a function of complex refractive index $m=(n,k)$ and the maximum wavelength λ_{max} in the band is approximately the same result obtained using the theory of Anomalous Diffraction[1] for complex refractive indices near 1 where the low frequency drop in Q occurs around a phase shift of π through the sphere diameter yielding the following result

$$\mathrm{D_{opt}} \approx \lambda_{max}/(2|\mathrm{m}-1|)$$

This indicates that in the limit of small refractive index the optimum complex refractive index is the largest achievable since that minimizes D_{opt} and maximizes α. For refractive indices that are not small Mie calculations show that D_{opt} is close to the result obtained at the intersection of the low frequency Rayleigh theory[1,2] and high frequency geometric optics plus diffraction scattering theories.[1,2] Setting the high frequency extinction cross section on the left hand side of the following equation equal to the low frequency Rayleigh absorption plus scatter cross sections on the right had side we get

$$2\pi\left(\frac{D}{2}\right)^2 = \left(\frac{2\pi}{\lambda}\right)4\pi\left(\frac{D}{2}\right)^3 \mathrm{Im}\left[\frac{m^2-1}{m^2+2}\right] + \frac{8}{3}\pi\left(\frac{D}{2}\right)^2\left(\frac{\pi D}{\lambda}\right)^4\left|\frac{m^2-1}{m^2+2}\right|^2$$

for an absorbing and scattering sphere. For a metal with little absorption we drop the absorption term (leading term on the right hand side of the previous equation) and add a magnetic dipole scatter term equal to ¼ the electric dipole scatter term yielding the equation

$$2\pi\left(\frac{D}{2}\right)^2 = \left(\frac{5}{4}\right)\frac{8}{3}\pi\left(\frac{D}{2}\right)^2\left(\frac{\pi D}{\lambda}\right)^4\left|\frac{m^2-1}{m^2+2}\right|^2$$

Solving this equation for D_{opt} we have

$$\mathrm{D_{opt}} \approx \left(\frac{3}{5}\left|\frac{m^2+2}{m^2-1}\right|^2\right)^{1/4}\frac{\lambda}{\pi}$$

Once again in order to minimize D_{opt} to maximize α, we want the largest achievable complex refractive index. Dielectric materials such as titanium dioxide and metals at visible and longer wavelengths have high refractive indices and achieve maximum spectral

extinction coefficients of $\alpha_{max} \approx 9 / \lambda_{max}$. Therefore in the visible spectral region where λ_{max}=0.7μ we have $\alpha_{max} \approx$ 13 m^2/cc and in the infrared where λ_{max}=14μ we have $\alpha_{max} \approx$ 0.64 m^2/cc . Conductive foam and bubbles can increase mass extinction coefficients by decreasing density but the only way to increase both mass α[m^2/g] and volume α[m^2/cc] extinction coefficients in the infrared is to use conductive fibers or flakes.

3.2 Calculating Low Frequency Cross Sections for Conductive Rods and Disks

Non-spherical particle scattering theories[4-20] such as the Separation of Variables Method for spheroids, the Extended Boundary Condition Method, Discrete Dipole Approximation and others are not suited to compute extinction cross sections of high aspect ratio prolate and oblate spheroidal shaped particles with large metallic refractive indices because of instabilities in the solutions and long computation times. Instead, electromagnetic extinction cross sections of metal disks are rapidly computed using a combination of diffraction, thin film optics and Rayleigh oblate finite and perfect conducting spheroid scattering theories. Electromagnetic extinction cross sections of metal rods are rapidly computed using a combination of infinite cylinder and Rayleigh prolate finite and perfect conducting spheroid scattering theories. These theories allow computation of the extinction efficiency factor Q for polydisperse and randomly oriented rods and disks over a wide wavelength range. Disk and rod aerosol size, aspect ratio and orientation distributions average out extinction cross section resonances permitting disks to represent lamellar or flake shapes and permitting rods to represent filamentary or fiber shapes leading to the following simplified analysis. As mentioned before, Q depends upon particle dimensions and complex refractive index which is a function of wavelength and, in the case of very small particles, also a function of particle dimensions which limit the conduction electron mean free path. Complex refractive index as a function of wavelength is predicted using the Drude model along with a thin film model for boundary reduction of conduction electron mean free path in the case of disks and along with a wire model for boundary reduction of conduction electron mean free path in the case of rods. In the analysis the orientation distribution is assumed to be random as is the case when Brownian and turbulent diffusion forces dominate over preferred orientation forces due to air drag acting against gravity.

We have the following result for low frequency Rayleigh absorption and scattering cross sections of randomly oriented ellipsoids[1, 2] with semi-axes a, b and c

$$\langle C_{abs} \rangle = k\,\mathrm{Im}\left\{ \frac{1}{3}\alpha_1 + \frac{1}{3}\alpha_2 + \frac{1}{3}\alpha_3 \right\}$$

$$\langle C_{sca} \rangle = \frac{k^4}{6\pi}\left\{ \frac{1}{3}|\alpha_1|^2 + \frac{1}{3}|\alpha_2|^2 + \frac{1}{3}|\alpha_3|^2 \right\}$$

where $k = \dfrac{2\pi}{\lambda}$ and the polarizability α_i along semi-axis *i*,

$$\alpha_i = 4\pi abc \frac{\varepsilon_1 - \varepsilon_m}{3\varepsilon_m + 3L_i(\varepsilon_1 - \varepsilon_m)},$$

is defined in terms of the complex permittivity of the spheroid ε_1, that of the medium ε_m, and the depolarization factor L_i along semi-axis *i*.

The complex refractive is equal to the square root of the complex permittivity $m = \sqrt{\varepsilon}$ and the permittivity of air is close to unity $\varepsilon_m = 1$. For a prolate spheroid the depolarization factor along the symmetry axis is

$$L_1 = \frac{1-e^2}{e^2}\left(-1+\frac{1}{2e}\ln\frac{1+e}{1-e}\right) \qquad e^2 = 1-\frac{b^2}{a^2}$$

and along the equatorial axes the depolarization factors are

$$L_2 = (1-L_1)/2$$

$$L_3 = L_2$$

For an oblate spheroid the depolarization factors along the equatorial axes are

$$L_1 = \frac{g(e)}{2e^2}\left[\frac{\pi}{2} - \tan^{-1} g(e)\right] - \frac{g^2(e)}{2}$$

$$g(e) = \left(\frac{1-e^2}{e^2}\right)^{1/2} \qquad e^2 = 1-\frac{c^2}{a^2}$$

$$L_2 = L_1$$

and along the symmetry axis the depolarization factor is

$$L_3 = 1-2L_1$$

These expressions originate from the Rayleigh theory of scattering by ellipsoidal shapes that create uniform internal electric (polarization) fields in a uniform external electric field. Prolate spheroids can be used to approximate rods and the oblate spheroids can approximate disks. This theory is applicable as long as the wavelength both inside and outside the particle is large compared to the particle dimensions. Writing rod and disk major dimension as H and minor dimension as h, this means that $H \ll \lambda$ and even more restrictively that $|mH| \ll \lambda$ in the case of metal rods and disks at infrared and longer wavelengths where m can be large. We often expect the condition on mH to be violated, however the Rayleigh electric dipole results above stand because, unlike spheres, the low frequency extinction cross section correction due to the magnetic dipole contribution is negligible compared to the electric dipole contribution for rods and disks[1].

3.3 Calculating High and Intermediate Frequency Extinction Cross Sections for Conductive Rods

At wavelengths on the order of and smaller than the major dimension we use the infinite cylinder solution for rods and the physical optics plus diffraction result for disks. Actually, the solution giving the smallest value for Q is chosen at wavelengths above and below the intersection of the Rayleigh theories with these theories. Either Rayleigh or infinite cylinder is chosen in the case of rods and either Rayleigh or physical optics plus diffraction is chosen in

the case of disks on either side of the intersection of these two theories which occurs around $\lambda \approx 3H$ If a switch is made from low to intermediate/high frequency solutions at $\lambda \approx 3H$, there will be a discontinuity. This is fixed with a bridging formula

$$\alpha = \left(\frac{1}{\left(\frac{1}{\alpha_{\mathrm{L}}}\right)^{n} + \left(\frac{1}{\alpha_{\mathrm{IH}}}\right)^{n}} \right)^{1/n}$$

where n □ 1, α_{L} is the low frequency and α_{IH} is the intermediate/high frequency extinction coefficient. I have chosen simply to pick the smallest of the two at any wavelength which is equivalent to $n = \infty$ and results in discontinuities in the derivatives.

The solution for radiation incident upon an infinite cylinder at an oblique angle of incidence is known but I have chosen to use the computationally less time consuming result for an infinite cylinder illuminated normal to its axis since the cross section for an orientation orthogonal to this plane is negligible. In this case the simple closed form solution for the extinction cross section for unpolarized radiation incident normal to the cylinder axis can be used to represent the extinction cross section averaged over random orientation in the plane normal to the Poynting vector. The scatter and extinction cross sections for an incident electric field polarized in the plane defined by the incident Poynting vector and the cylinder axis given by[2]

$$Q_{\mathrm{sca,\,I}} = \frac{W_{sca,\mathrm{I}}}{2aLI_i} = \frac{2}{x}\left[|b_{0\mathrm{I}}|^2 + 2\sum_{n=1}^{\infty}\left(|b_{n\mathrm{I}}|^2 + |a_{n\mathrm{I}}|^2\right) \right]$$

$$Q_{\mathrm{ext,\,I}} = \frac{W_{ext,\mathrm{I}}}{2aLI_i} = \frac{2}{x}\mathrm{Re}\left\{ b_{0\mathrm{I}} + 2\sum_{n=1}^{\infty} b_{n\mathrm{I}} \right\}$$

and the scatter and extinction cross sections for an incident electric field polarized perpendicular to that plane are

$$Q_{\mathrm{sca,\,II}} = \frac{2}{x}\left[|a_{0\mathrm{II}}|^2 + 2\sum_{n=1}^{\infty}\left(|a_{n\mathrm{II}}|^2 + |b_{n\mathrm{II}}|^2\right) \right]$$

$$Q_{\mathrm{ext,\,II}} = \frac{2}{x}\mathrm{Re}\left\{ a_{0\mathrm{II}} + 2\sum_{n=1}^{\infty} a_{n\mathrm{II}} \right\}$$

and for unpolarized incident fields we have

$$Q_{\mathrm{sca}} = \frac{1}{2}\left(Q_{\mathrm{sca,I}} + Q_{\mathrm{sca,II}}\right), \qquad Q_{\mathrm{ext}} = \frac{1}{2}\left(Q_{\mathrm{ext,II}} + Q_{\mathrm{ext,I}}\right)$$

where

$$a_{nI} = \frac{C_n V_n - B_n D_n}{W_n V_n + iD_n^2}, \qquad b_{nI} = -\frac{W_n B_n + iD_n C_n}{W_n V_n + iD_n^2}$$

$$D_n = n\cos\varsigma \cdot \eta J_n(\eta) H_n^{(1)}(\xi)\left(\frac{\xi^2}{\eta^2} - 1\right)$$

$$H^{(1)} = J + iY$$

$$B_n = \xi\left[m^2 \xi J_n'(\eta) J_n(\xi) - \eta J_n(\eta) J_n'(\xi)\right]$$

$$C_n = n\cos\varsigma\eta J_n(\eta) J_n(\xi)\left(\frac{\xi^2}{\eta^2} - 1\right)$$

$$V_n = \xi\left[m^2 \xi J_n'(\eta) H_n^{(1)}(\xi) - \eta J_n(\eta) H_n^{(1)\prime}(\xi)\right]$$

$$W_n = i\xi\left[\eta J_n(\eta) H_n^{(1)\prime}(\xi) - \xi J_n'(\eta) H_n'(\xi)\right]$$

$$b_{nI}\left(\varsigma = 90^\circ\right) = b_n = \frac{J_n(mx) J_n'(x) - mJ_n'(mx) J_n(x)}{J_n(mx) H_n^{(1)\prime}(x) - mJ_n'(mx) H_n^{(1)}(x)}$$

$$a_{nII} = -\frac{A_n V_n - iC_n D_n}{W_n V_n + iD_n^2}, \qquad b_{nII} = i\frac{C_n W_n + A_n D_n}{W_n V_n + iD_n^2}$$

$$A_n = i\xi\left[\xi J_n'(\eta) J_n(\xi) - \eta J_n(\eta) J_n'(\xi)_n\right]$$

$$a_{nII}\left(\varsigma = 90^\circ\right) = a_n = \frac{mJ_n'(x) J_n(mx) - J_n(x) J_n'(mx)}{mJ_n(mx) H_n^{(1)\prime}(x) - mJ_n'(mx) H_n^{(1)}(x)}$$

For computational purposes it is better to write a_n and b_n in terms of the log derivative

$$D_n = \frac{J_n'(\rho)}{J_n(\rho)}$$

noting that for Bessel functions $Z = J$ or $Z = Y$

$$Z_n' = Z_{n-1}(x) - \frac{n}{x} Z_n(x)$$

thus

$$a_n = \frac{[D_n(mx)/m + n/x]J_n(x) - J_{n-1}(x)}{[D_n(mx)/m + n/x]H_n^{(1)}(x) - H_{n-1}^{(1)}(x)}$$

$$b_n = \frac{[mD_n(mx) + n/x]J_n(x) - J_{n-1}(x)}{[mD_n(mx) + n/x]H_n^{(1)}(x) - H_{n-1}^{(1)}(x)}$$

3.4 Calculating High and Intermediate Frequency Extinction Cross Sections for Conductive Disks

The extinction cross section of a thin rectangular plate having major dimensions on the order of or larger than the wavelength are computed based on Physical Optics and Diffraction theories. A plate of area A illuminated at an angle θ with respect to the normal to the plate surface will have the following extinction cross section[21]

$$C_E = 2A\cos\theta\left(1 - T(\theta)\right)$$

and absorption cross section

$$C_A = A\cos\theta\left(1 - T(\theta) - R(\theta)\right)$$

where disk reflectance $R(\theta)$ and transmittance $T(\theta)$ are those for an infinite slab[22] illuminated at an angle of incidence θ

$$R(\theta) = \left|\frac{i(Z_1^2 - Z_2^2)\tan a_2}{2Z_1Z_2\cos a_2 - i(Z_1^2 - Z_2^2)\tan a_2}\right|^2$$

$$T(\theta) = \left|\frac{2Z_1Z_2}{2Z_1Z_2\cos a_2 - i(Z_1^2 - Z_2^2)\tan a_2}\right|^2$$

where

$$a_2 = \frac{2\pi h\varepsilon^{1/2}}{\lambda}\left(1 - \frac{\sin^2\theta}{\varepsilon}\right)^{1/2}$$

$$Z_1 = \frac{1}{2}\left(\frac{1}{\cos\theta} + \cos\theta\right)$$

$$Z_2 = \frac{1}{\varepsilon^{1/2}}\left[\left(\frac{1}{1 - \frac{\sin^2\theta}{\varepsilon}}\right)^{1/2} + \left(1 - \frac{\sin^2\theta}{\varepsilon}\right)^{1/2}\right]$$

and ε is the complex permittivity of the slab. These disk cross sections must be integrated numerically over angle to obtain the cross section averaged over an ensemble of randomly oriented disks.

3.5 Calculating Conductivity and Permittivity of Metals as a Function of Frequency Using the Drude Model

The Drude model for complex refractive index can be used with reasonable accuracy for metals at infrared and longer wavelengths[23-27]. First the complex conductivity is written as function of DC conductivity σ, frequency ω, and conduction electron relaxation time τ.

$$\sigma(\omega) = \sigma/(1 - i\omega\tau)$$

and the complex permittivity is written

$$\varepsilon(\omega) = 1 + 4\pi i \sigma(\omega)/\omega$$

so that the complex refractive index for nonmagnetic materials becomes

$$\mathrm{m}(\omega) = \varepsilon(\omega)^{1/2}$$

3.6 Calculating Size Effects on Conductivity of Thin Metal Wires and Films

The effects of rod boundaries on rod complex refractive index are predicted by a theory[28, 29] for thin wire conduction electron scatter. Wire boundaries reduce DC conductivity σ from its bulk value σ_o because of a shorter conduction electron relaxation time as indicated by the following expression

$$\frac{\sigma}{\sigma_o} = 1/(1 + 1/\kappa)$$

where $\kappa = \dfrac{h}{\Lambda}$ is the ratio of wire diameter to conduction electron mean free path Λ in the bulk.

The effects of disk boundaries on complex refractive index is given by a theory[28, 30] for thin film conduction electron scatter. Now the film conductivity ratio becomes

$$\frac{\sigma}{\sigma_o} = 1 - \frac{3}{4}\left(\kappa - \frac{\kappa^3}{12}\right)E_i - (-\kappa) - \frac{3}{8\kappa}\left(1 - e^{-\kappa}\right) - \left(\frac{5}{8} + \frac{\kappa}{16} - \frac{\kappa^2}{16}\right)e^{-\kappa}$$

where κ is the ratio of film thickness to conduction electron mean free path in the bulk and E_i is the exponential integral

$$-E_i(-u)) = \int_u^\infty \frac{e^{-t}}{t}\mathrm{d}t$$

For film thicknesses less than a few nanometers quantum corrections may be necessary but these would involve experimental measurements to determine two parameters in the theory for each metal[31, 32].

4. DISCUSSION OF RESULTS AND FIGURES FOR DISKS AND RODS

4.1 Simple Result for Some Disks

For thin (h>>H) disks opaque to the radiation with diameters greater than about one third the wavelength the extinction efficiency factor is equal to two and the surface area to volume ratio is 2/h so the extinction coefficient may be written simply as

$$\alpha[\mathrm{m}^2/\mathrm{cc}] = 1/\mathrm{h}[\mu]$$

This result overestimates the extinction cross sections of disks that are partially transparent and disks with major dimensions less than about one third the wavelength. This might be interpreted as an upper limit on the extinction coefficient of a disk but if the transmittance *T(θ)* drops off less rapidly than 1/h it is not actually an upper limit.

4.2 Discussion of Size Effects on Complex Refractive Index

Complex refractive indices of metals in bulk form where particle size is large enough so that bounding surfaces have negligible effect on conduction electron mean free path and relaxation time, have been computed in the infrared based on the Drude model as a function of wavelength and DC conductivity. Results for the real and imaginary parts of the complex refractive indices appear in Figures 1 and 2 respectively using a typical value of $\mathbf{r_s/a_0}$=2.5 ; the radius of a sphere containing one conduction electron $\mathbf{r_s}$ in units of the Bohr radius $\mathbf{a_0}$, 0.529 x 10^{-8} cm. This then sets the Fermi velocity[23]

$$\mathrm{V_f} = 4.2\mathrm{x}10^8 / r_s\mathrm{a_0}[\mathrm{cm/sec}]$$

the conduction electron relaxation time

$$\tau = 0.22 r_s \mathrm{a_0}^3 \sigma \mathrm{x}10^{-14}[\mathrm{sec}]$$

and the conduction electron mean free path

$$\Lambda = \mathrm{V_f}\tau$$

where σ is the DC conductivity. When specifying DC conductivity σ [mho/cm] we must convert[26] to a set of units consistent with the Drude model expressions using

$$\sigma\left[\sec^{-1}\right] = \sigma[\mathrm{mho/cm}]\mathrm{c}^2[\mathrm{cm/sec}]\mathrm{x}10^{-9}$$

where c is the speed of light, 3 x 10^{10} cm/sec.

Figures 1 and 2 show the real and imaginary parts of metallic refractive indices in the bulk form as a function of wavelength and conductivity. Rod (thin wire) and disk (thin film)

size effects on complex refractive indices are shown in Figures 3-22 as a function of minimum dimension, conductivity and wavelength. Significant differences are evident between the infrared complex refractive indices in bulk form versus both thin wire and film forms where, even up to one micrometer minimum dimension, boundaries reduce conduction electron mean free path. Because extinction cross sections of metal rods and disks depend not only on particle major dimension, minor dimension, and wavelength but also strongly on the complex refractive index there will be significant differences in these cross sections.

Particle boundary reductions in conduction electron mean free path make the complex refractive index a function of particle size as well as conductivity and wavelength. Figures 3 and 4 show significant reductions in both real and imaginary parts of the complex refractive index for 1 nm diameter wires below the bulk values shown in the previous figures. Reductions in complex refractive indices for 1 nm thick films shown in Figures 5 and 6 are almost as great. Figures 7 and 8 display complex refractive indices for wires with a conductivity of 6×10^5 mho/cm, that of copper, over a range of diameters between 0.1nm and 1000nm. Similarly Figures 9 and 10, 11 and 12, 13 and 14 show this for wires at conductivities of 10^5 mho/cm, 10^4 mho/cm and 10^3 mho/cm (that of graphite) respectively. At the higher conductivities size effects are apparent up to wire diameters approaching 100nm. Figures 15 and 16, 17 and 18, 19 and 20, 21 and 22 show complex refractive indices for films with conductivities of 6×10^5 mho/cm, 10^5 mho/cm, 10^4 mho/cm and 10^3 mho/cm respectively over a thickness range from 0.1nm to 1000nm. Size reductions in film complex refractive indices below the bulk value are evident up to thicknesses approaching 100nm at the higher conductivities.

4.3 Discussion of Rod Infrared Spectral Extinction Coefficients

Incorporating the size effects just mentioned into calculations of spectral extinction cross sections of infinite cylinders and Rayleigh cylinders make it possible to accurately predict extinction cross sections for cylinders with diameters less than 100nm. By combining the infinite cylinder solution with the Rayleigh solution for high aspect ratio prolate spheroids we are in effect combining a high frequency solution for rod lengths much greater than the wavelength with a low frequency solution for rod lengths much less than the wavelength. In this way the entire extinction cross section spectra of metal rods are covered as long as extinction cross section resonances in the region where rod lengths are on the order of the wavelength get washed out by averaging length, orientation and diameter distributions. This should occur for sufficiently broad distributions and large populations of metal rods as would be found for example in smoke clouds. Rod lengths H and diameters h appearing in the figures should be interpreted as representative of the length and diameter population distributions.

Rod extinction coefficients are shown in Figures 23-68 as a function of wavelength, diameter, length and conductivity. Rod length and conductivity are held fixed at levels of interest in Figures 23-38. Rod length and diameter are held fixed in Figures 39-48, diameter and conductivity are held fixed in Figures 49-60 and wavelength and length are held fixed in Figures 61-68. Extinction coefficient is maximized (~800m^2/cc at a wavelength of 14 micrometers) at an optimum combination of complex refractive index that is the largest achievable (that of copper; $6\text{x}10^5$mho/cm), any length greater than about one third the wavelength and a diameter of about 16nm. Extinction coefficients nearly that large (~600m^2/cc at a wavelength of 14 micrometers) are possible for metals such as iron with a conductivity of 10^5mho/cm at lengths greater than about one third the wavelength and slightly larger diameters. If lengths that large are not achievable then extinction coefficients can be maximized at lower levels using optimum combinations of length, conductivity and diameter at various wavelengths read from the appropriate figure.

4.4 Discussion of Disk Infrared Spectral Extinction Coefficients

We assume that a large population of metal disks are attenuating infrared radiation and that the extinction cross section is averaged over sufficiently broad diameter, orientation and thickness distributions as would be encountered for example in clouds of metal flake smoke. In this way extinction cross section resonances are wiped out in the region where disk diameter is on the order of the wavelength and physical optics plus diffraction used to compute extinction cross sections of disks with diameters large compared to the wavelength is extended up to wavelengths on the order of the diameter. Likewise the Rayleigh solution for oblate spheroid scattering used to compute extinction cross sections of disks with diameters small compared to the wavelength is extended down to wavelengths on the order of the diameter. After incorporating the Drude model along with a model for conduction electron mean free path limitations due to thin film boundaries the full extinction cross section spectra of metal disks in the infrared are covered by a combination of these high and low frequency theories. Generally the low and high frequency theories are found to meet at $\lambda \approx 3D$. Disk diameters H and thicknesses h appearing in the figures should be interpreted as representative of the diameter and thickness population distributions.

Disk extinction coefficients are shown in Figures 69-107 as a function of wavelength, diameter, thickness and conductivity. Disk diameter and thickness are held fixed at levels of interest in Figures 69-77. Disk diameter and conductivity are held fixed in Figures 78-93, thickness and conductivity are held fixed in Figures 94-99 and wavelength and diameter are held fixed in Figures 100-107. Extinction coefficient is maximized (~$350m^2$/cc at a wavelength of 14 micrometers) at an optimum combination of complex refractive index that is the largest achievable, any diameter greater than about one third the wavelength and a thickness of about 1nm. Extinction coefficients nearly that large (~$250m^2$/cc at a wavelength of 14 micrometers) are possible for metals such as iron with a conductivity of 10^5mho/cm at diameters greater than about one third the wavelength and slightly larger thicknesses. If diameters that large are not achievable then extinction coefficients can be maximized at lower levels using optimum combinations of diameter, conductivity and thickness at various wavelengths read from the appropriate figure.

4.5 Metal Coated Rods and Disks

Rough estimates of extinction coefficients of metal coated dielectric rods and disks can be made by reducing the previously computed extinction coefficients by the volume fraction of the dielectric substrate material. In the following plots this was done assuming a metal rod or disk equal in major dimension to that of the substrate material with a volume equal to the coating volume. Metal coated dielectric rod extinction coefficients computed in this way are shown in Figures 108 and 109 as a function of coated rod overall diameter and coating conductivity at a wavelength of 14 micrometers, length of 8 micrometers and dielectric substrate diameter of 20nm. Metal coated dielectric disk extinction coefficients are shown in Figures 110 and 111 as a function of coated disk overall thickness and coating conductivity at a wavelength of 14 micrometers, diameter of 7 micrometers and dielectric substrate thickness of 20nm. These figures are relevant because some available dielectric flake material such as mica and talc cleave preferentially along a plane that further reduces thickness during milling. Milling and then coating these particles may have advantages over reducing the thickness of a metal flake by using its malleability during milling or trying to produce thin free standing metal flakes.

4.6 Magnitude of Size Effects on Extinction Coefficients

Metal rod extinction coefficients were calculated as a function of diameter and length at a conductivity of 10^5mho/cm and a wavelength of 3μ in Figure 112 including size effects and in Figure 113 excluding size effects. Large decreases in the extinction coefficient due to size effects are evident for diameters under a few nanometers. A similar comparison was made at a wavelength of 14μ in Figures 114 and 115 and again extinction coefficients are dramatically reduced for diameters under a few micrometers.

Metal disk extinction coefficients were calculated as a function of thickness and diameter at a conductivity of 10^5mho/cm and a wavelength of 3μ in Figure 116 including size effects and in Figure 117 excluding size effects. Large decreases in the extinction coefficient due to size effects are evident for thicknesses under a few nanometers. A similar comparison was made at a wavelength of 14μ in Figures 118 and 119 and again extinction coefficients are dramatically reduced for thicknesses under a few micrometers.

4.7 Transition Frequency Errors Near the Merger of the High and Low Frequency Solutions

Low and high frequency extinction cross section solutions become more accurate the farther they are from transition or intermediate frequencies located where the solutions are merged using one of several bridging techniques. The bridging technique used throughout this report involves choosing the smaller extinction coefficient and results as a function of minimum and maximum dimensions appear in Figures 112 and 114 for rods and Figures 116 and 118 for disks. This approach preserves some of the artificiality of the construct by making all derivatives discontinuous while the function is continuous.

Comparable results involving choosing the low frequency solution for major dimension less than three times the wavelength appear in Figures 120 and 121 for rods and Figures 124 and 125 for disks. Here the function itself suffers a considerable discontinuity but the high and low frequency extremes are not contaminated by a bridging equation. Finally the reciprocal bridging equation was used in Figures 122 and 123 for rods and Figures 126 and 127 for disks. All derivatives are continuous and any evidence of the bridge has vanished.

5. CONCLUSIONS

Maximum sphere extinction coefficients in units of m^2/cm^3 are $\sim 9/\lambda$ where λ is the wavelength of interest expressed in micrometers (e.g., 0.64 m^2/cm^3 at $\lambda = 14$ micrometers). Optimum diameter and complex refractive index producing this maximum is a diameter $D \sim \lambda/3$ and the largest complex refractive index achievable typical of metals. It should be mentioned that the small sphere resonance at a complex refractive index of $\left(0,\sqrt{2}\right)$ gets washed out over any wavelength band just as the large real refractive index resonances get washed out due to size distribution.

The most dependable metal rod and disk extinction coefficient calculations are made away from the transition region where major dimension is on the order of the wavelength. Accurate low frequency results appear in many figures where the major dimension is less than about one tenth the wavelength. Accurate high frequency results appear near the back (major

dimension much greater than wavelength) face of Figures 120, 121, 124 and 125 where high frequency extinction coefficients are displayed at all major dimensions greater than one third the wavelength while low frequency extinction coefficients are displayed at major dimensions less than one third the wavelength. These four figures show that high frequency extinction coefficients reach maximum values of ~300m^2/cc and ~800m^2/cc at wavelengths of 3μ and 14μ respectively for rods and ~150m^2/cc at both wavelengths for disks. Optimum rod diameters are ~3nm and ~8nm at wavelengths of 3μ and 14μ respectively and optimum disk thicknesses are ~0.7nm at both wavelengths. Higher conductivity metals will have slightly higher maximum extinction coefficients and smaller optimum minimum dimensions and lower conductivity metals will have significantly lower maximum extinction coefficients at larger optimum minimum dimensions. Rods 300nm long and disks 300nm in diameter have maximum low frequency extinction coefficients that are almost as large with optimum minimum dimensions close to those just given for much larger major dimension rods and disks. Further reductions in rod and disk major dimensions reduce the maximum extinction coefficient and reduces the optimum minimum dimension.

There are essentially three types of errors or potential errors in the calculated extinction cross sections. First there are errors that result from extending low frequency calculations outside the region where wavelength is much greater than major dimension and extending high frequency calculations outside the region where wavelength is much less that major dimension. Unfortunately a great deal of interest is directed at the transition region where major dimension is on the order of the wavelength because extinction coefficients increase with increasing major dimension until major dimension is on the order of the wavelength. Thus optimum rod length and optimum disk diameter are in the transition region and no bridging approach can completely fix this problem. Second the size effects models are accurate down to minimum dimensions around 10nm but optimum rod diameters are in the 3-8nm range and optimum disk thicknesses are less than 1nm. Third the calculations assume that rods have a population distribution of lengths and disks have a distribution of diameters that average out resonances that would otherwise occur in the transition region.

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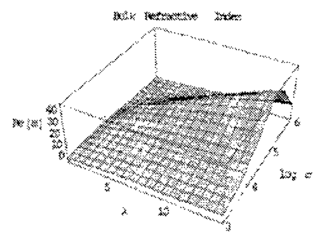
Figure 1



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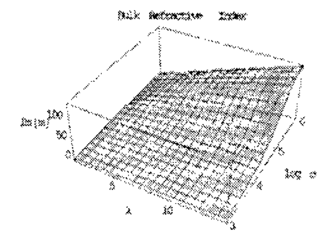

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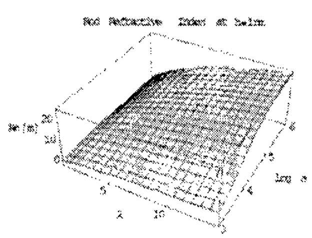

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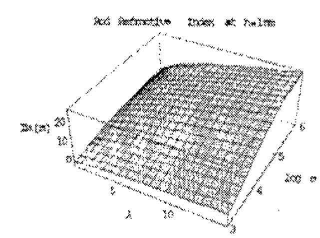

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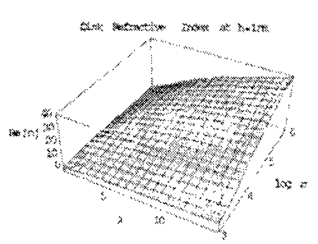

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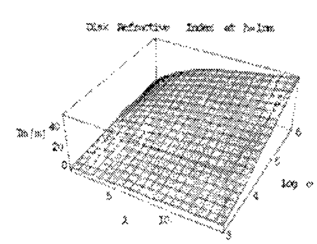

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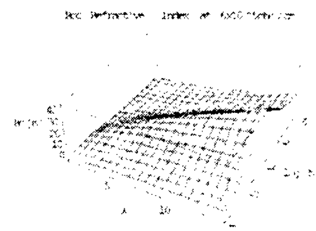

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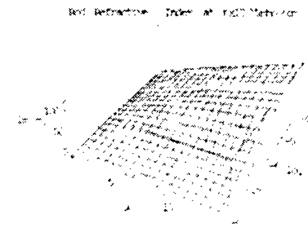

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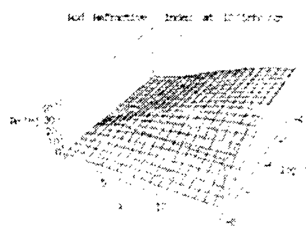

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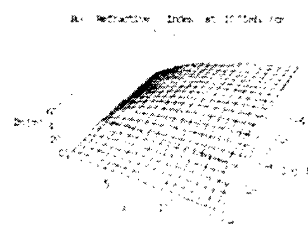

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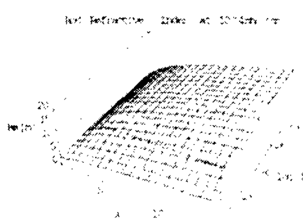

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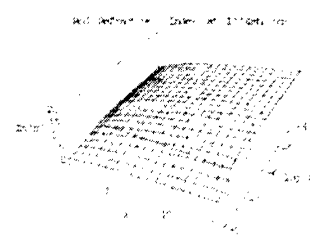

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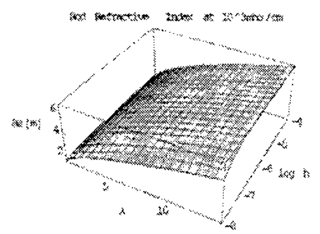

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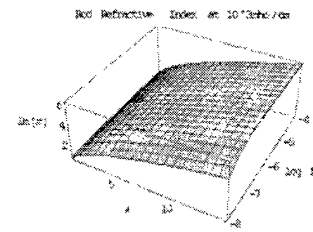

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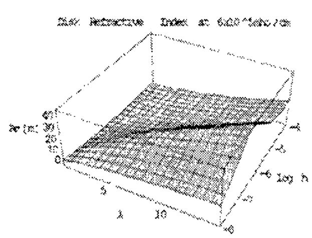

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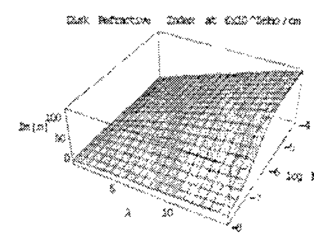

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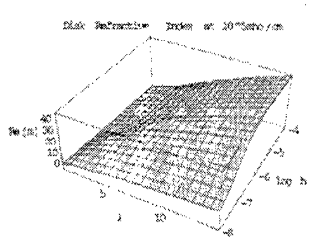

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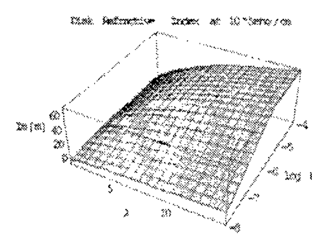

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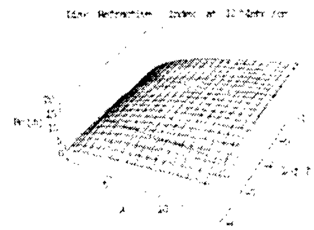

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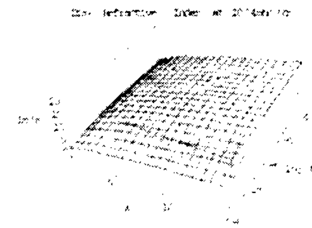

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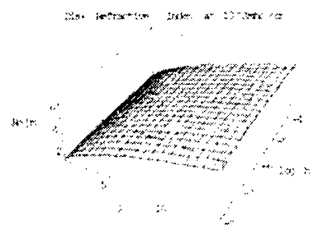

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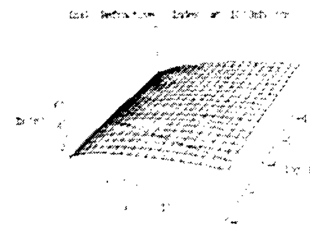

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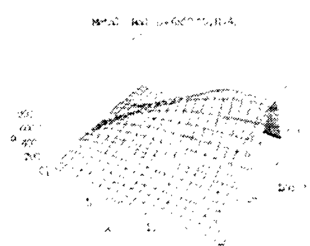

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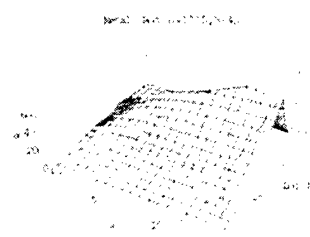

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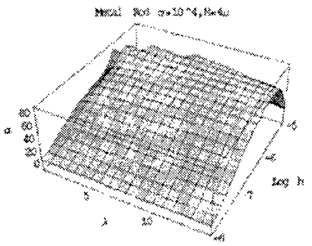

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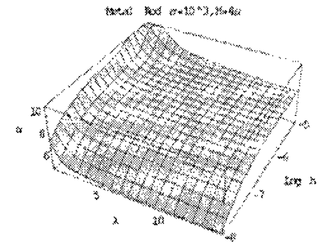

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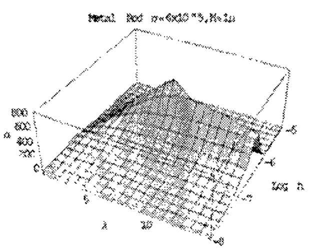

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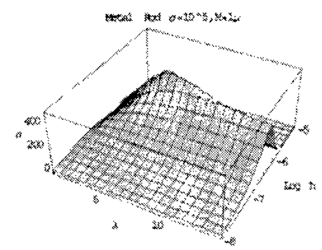

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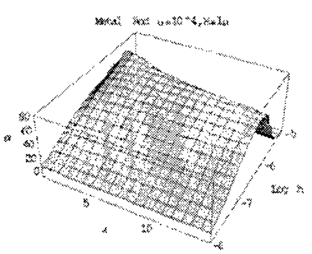

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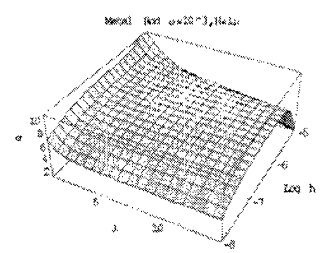

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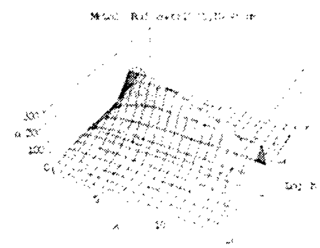

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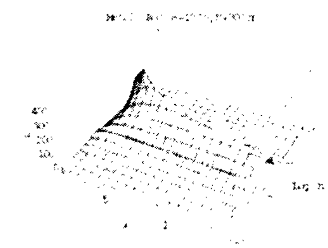

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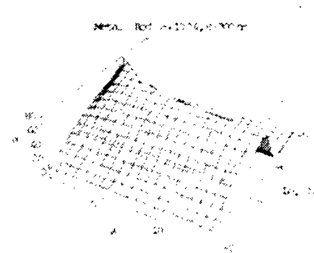

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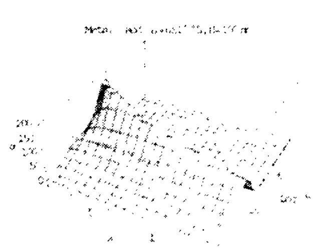

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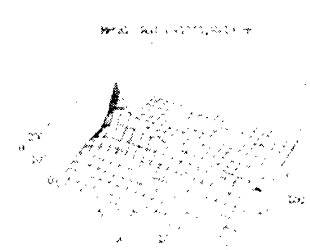

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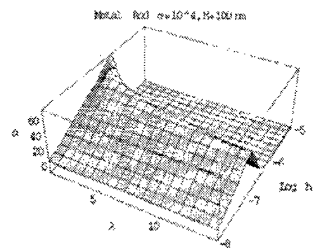

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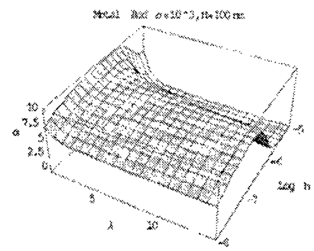

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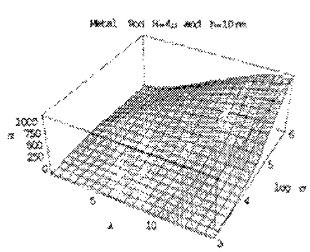

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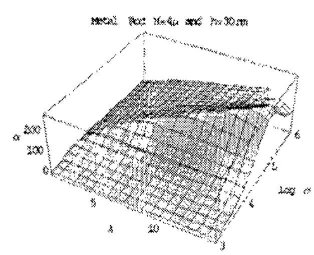

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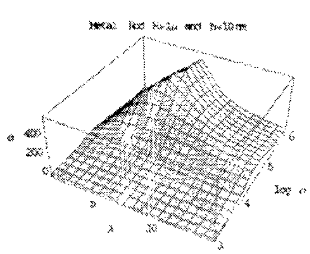

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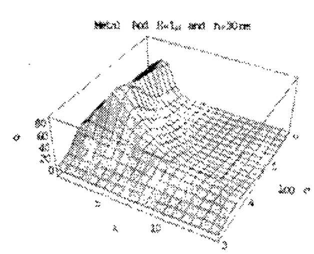

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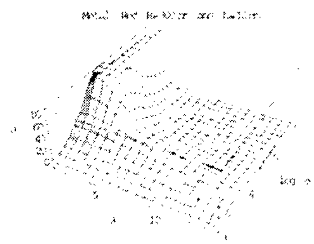

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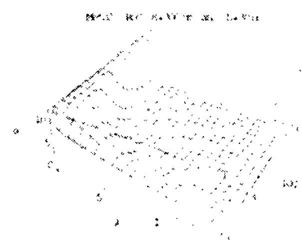

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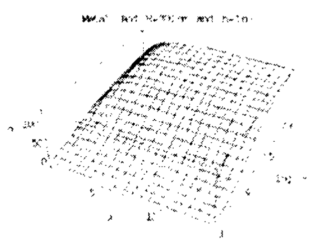

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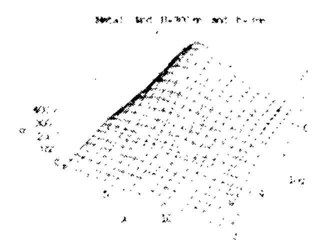

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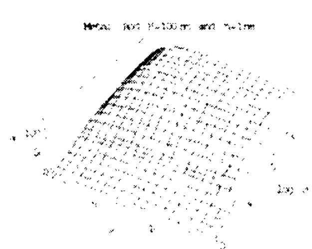

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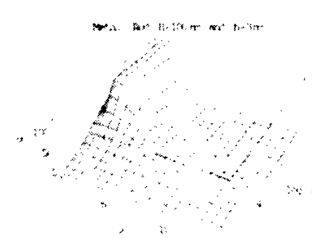

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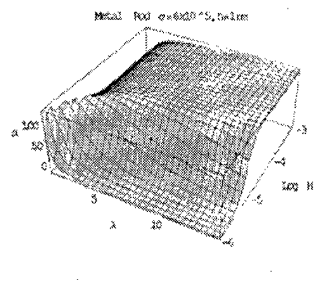

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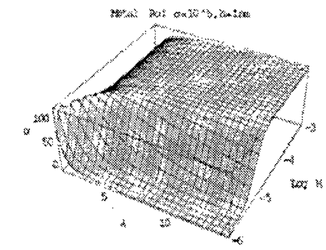

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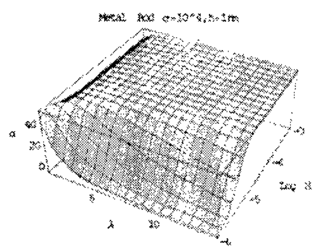

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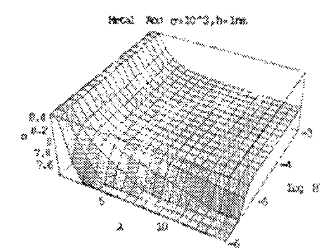

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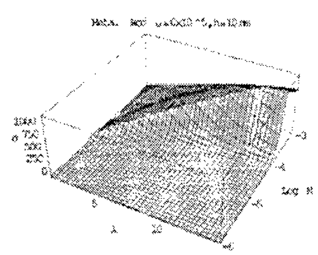

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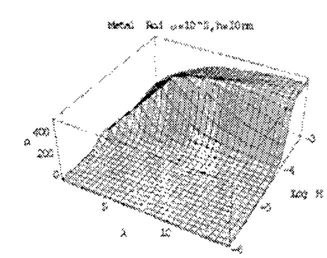

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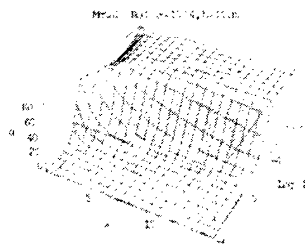

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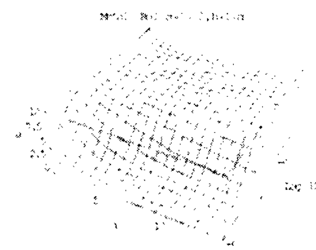

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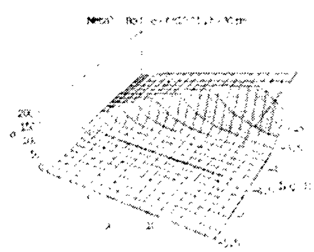

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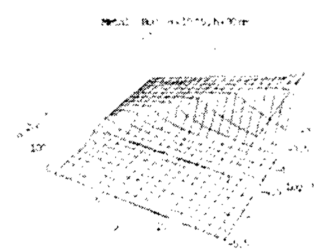

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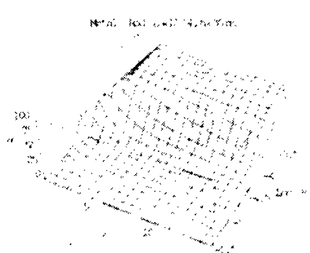

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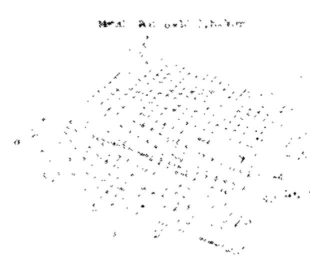

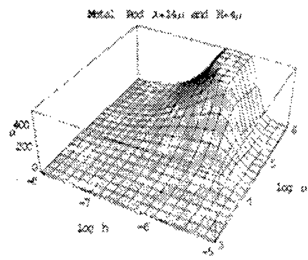
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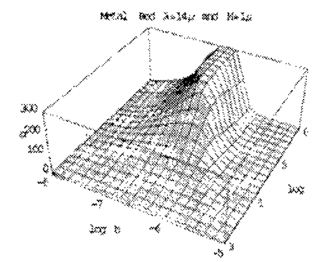
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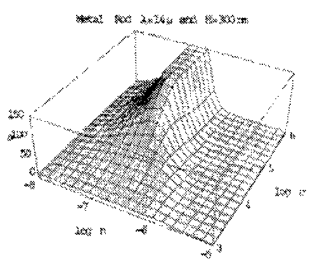
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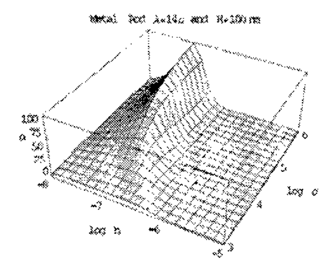
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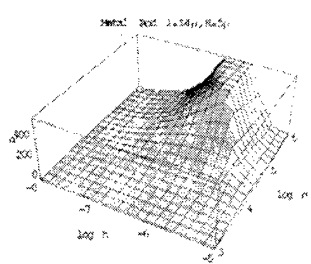
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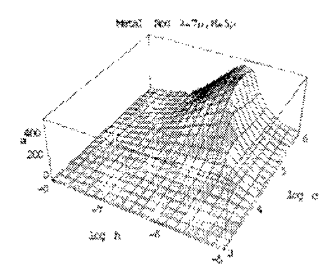
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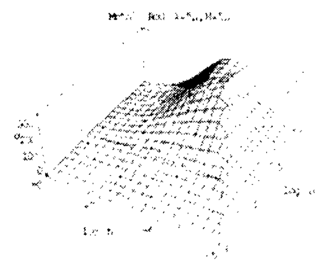

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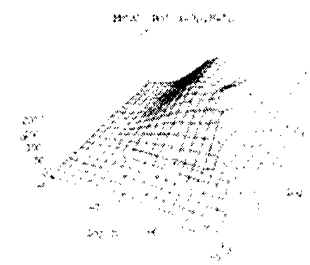

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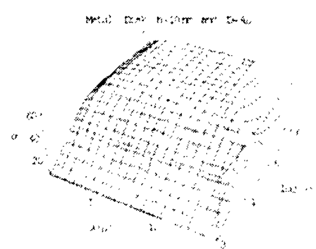

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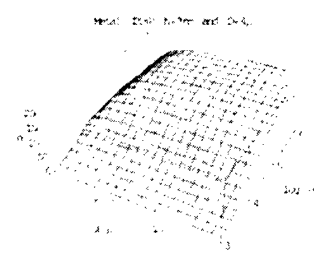

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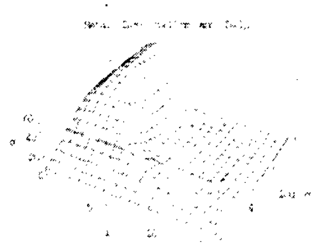

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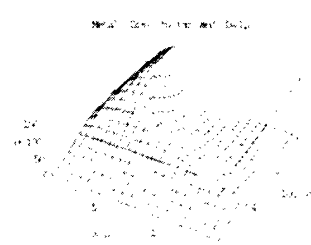

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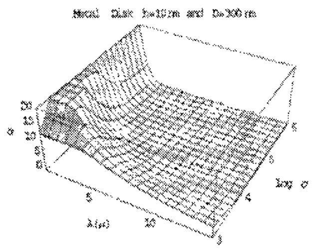

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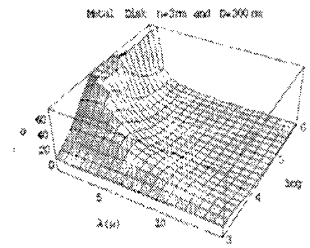

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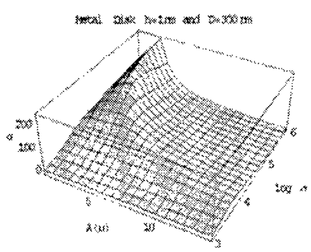

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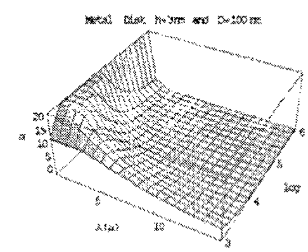

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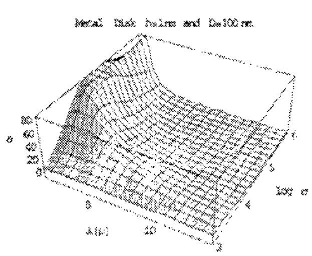

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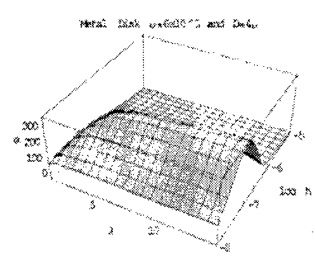

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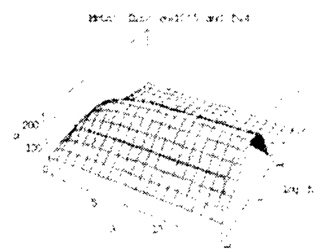

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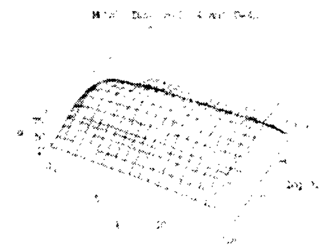

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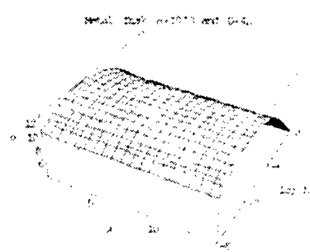

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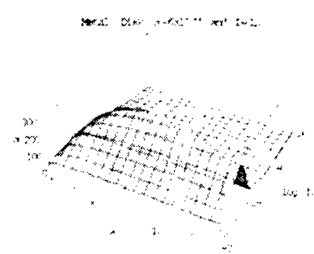

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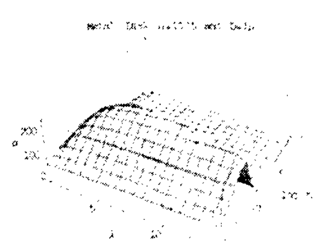

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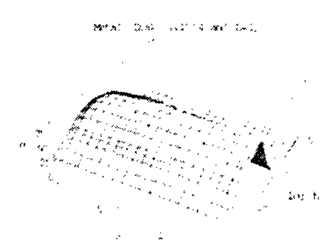

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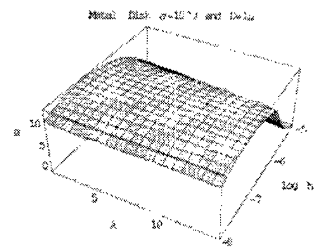

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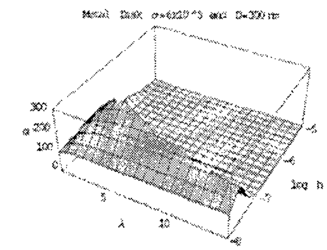

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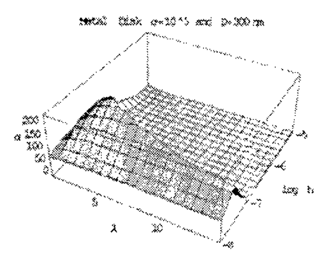

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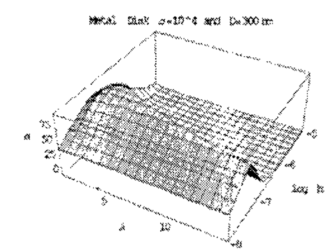

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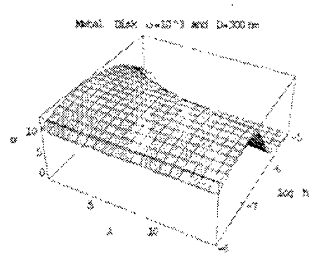

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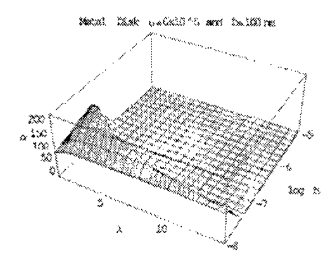

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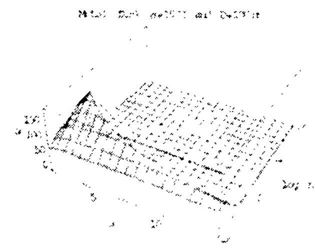

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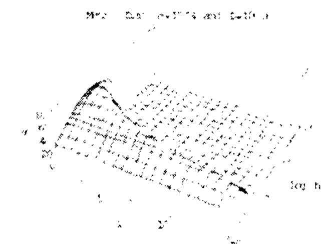

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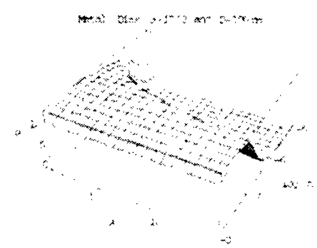

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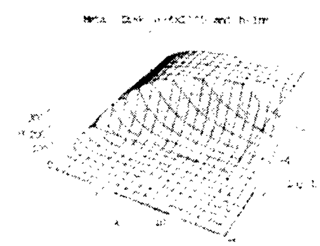

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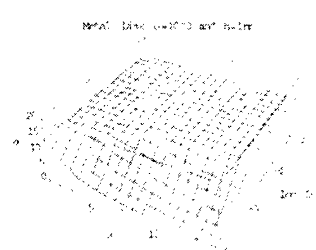

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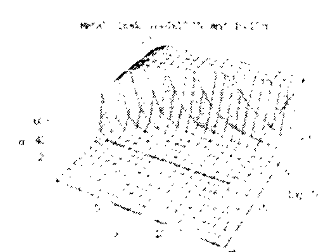

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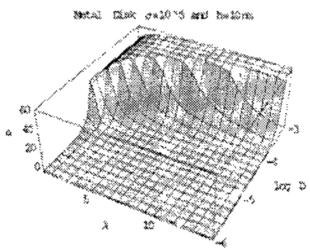

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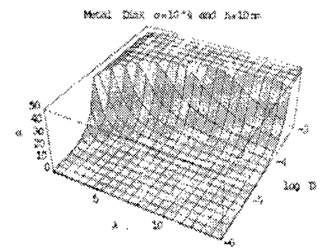

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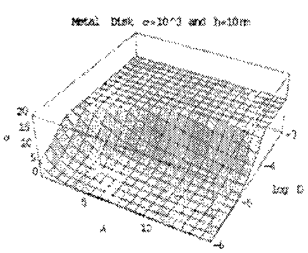

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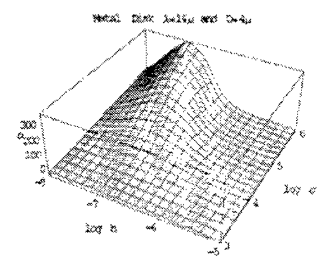

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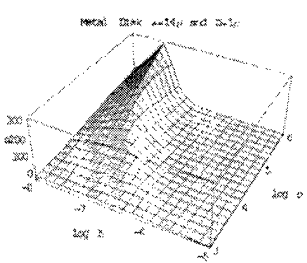

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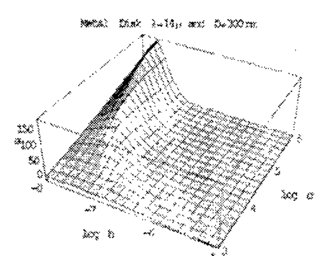

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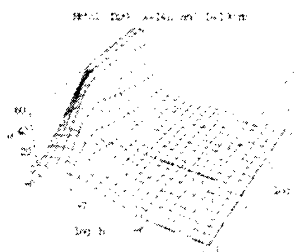

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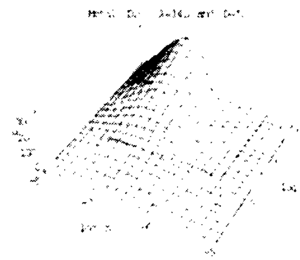

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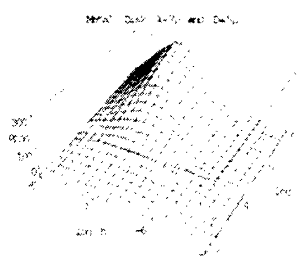

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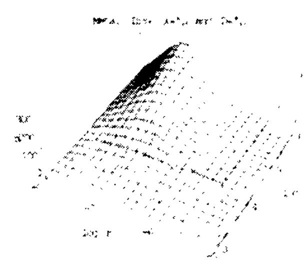

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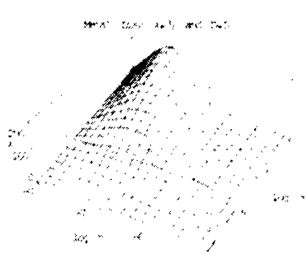

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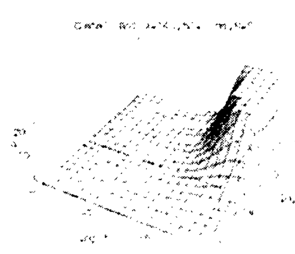

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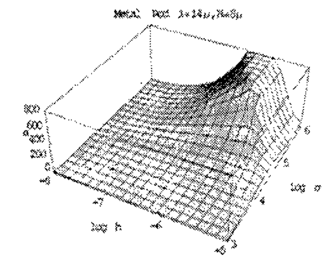

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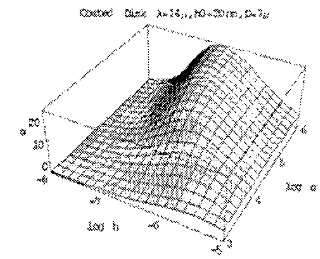

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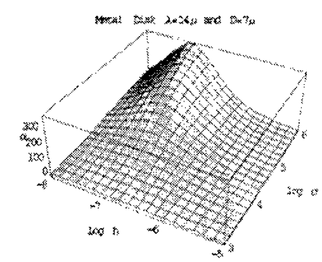

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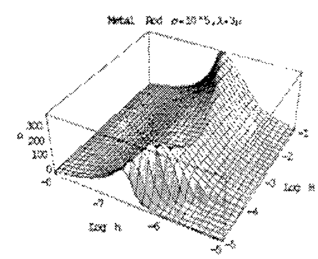

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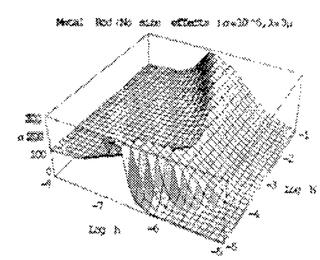

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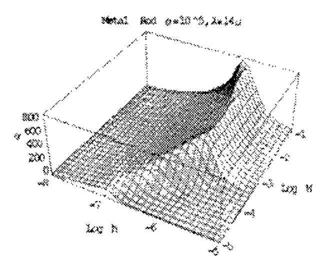

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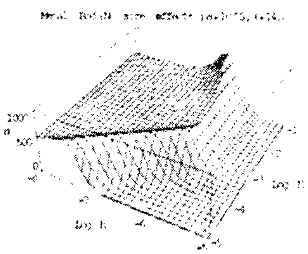

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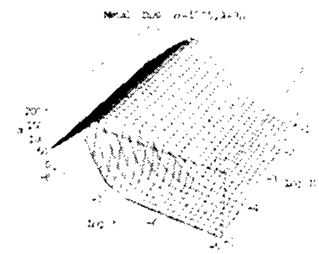

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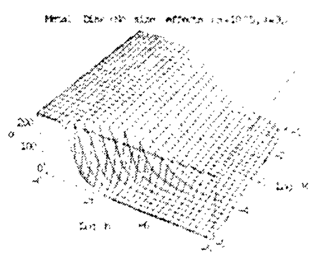

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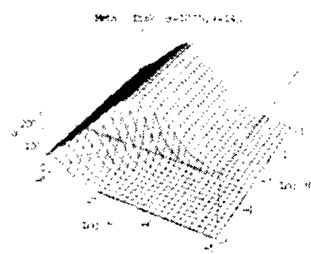

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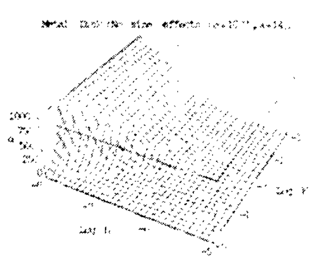

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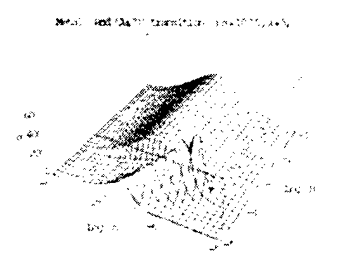

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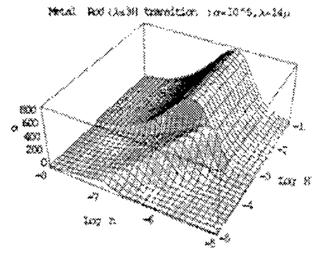

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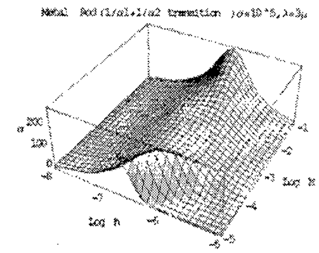


Figure 123

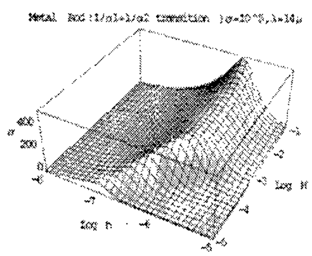


Figure 124

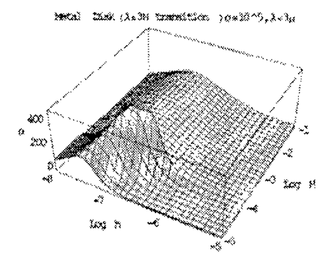


Figure 125

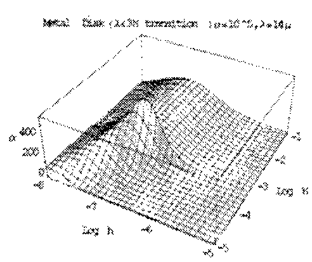


Figure 126

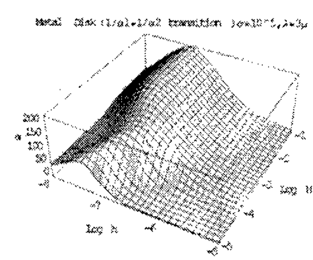
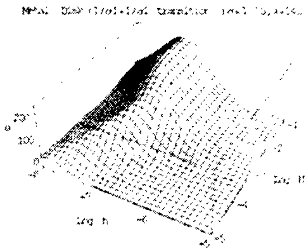


Figure 127



LITERATURE CITED

1. Van de Hulst, *Light Scattering by Small Particles*, Dover Publications, Inc., (1981).

2. Bohen, C.F. and Huffman, D.R., *Absorption and Scattering of Light by Small Particles*, John Wiley & Sons (1983).

3. Hovenier, J.W., Lumme, K., Mishchenko, M.I., Voshchinnikov, N.V., Mackowski, D.W., and Rahola, J., "Computations of Scattering Matrices of Four Types of Non-Spherical Particles Using Diverse Methods," J. Quant. Spectrosc. Radiat. Transfer, Vol. 55, No. 6, pp. 695-705 (1996).

4. Schuerman, D.W., ed., *Light Scattering by Irregularly Shaped Particles*, Plenum Press, New York, NY (1980).

5. Varadan, V.K. and Varadan, V.V., eds., *Acoustic, Electromagnetic and Elastic Wave Scattering – Focus on the T-Matrix Approach*, Pergamon Press, New York, NY, 1981.

6. Oguchi, T., Radio Sci., Vol 16, p. 691 (1981).

7. Wiscombe, W.J. and Mugnai, A., "Single Scattering from Non Spherical Chebyshev Particles: a Compendium of Calculations", NASA Ref. Publ. 1157, NASA/GSFC, Greenbelt, MD, Appendix A (1986).

8. Barber, P.W. and Hill, S.C., *Light Scattering by Particles: Computational Methods*, World Scientific, Singapore (1990).

9. Mishchenko, M.I., Travis, L.D., and Mackowski, D.W., JQSRT Vol. 55, p. 535 (1996).

10. Wait, J.R., Can. J. Phys. Vol. 33, p. 189 (1955).

11. Waterman, P.C., Phys. Rev. Vol. D3, p. 825 (1971).

12. Peterson, B. and Strom, S. Phys. Rev. Vol. D8, p. 3661 (1973).

13. Purcell, E.M. and Pennypacker, C.R., Astrophys. J. Vol. 186, p. 705 (1973).

14. Asano, S. and Yamamoto, G., Appl. Opt. Vol. 14, p. 29 (1975).

15. Farafonov, V.G., Differential Equations (Sov.) Vol. 19, p. 1765 (1983).

16. Draine, B.T., Astrophys. J., Vol. 333, p.848, (1988).

17. Fuller, K.A., Appl. Opt. Vol. 30, p. 4716 (1991).

18. Mishchenko, M.I., JOSA Vol. A8, p.871 (1991).

19. Kuik, F., de Haan, J.F., and Hovenier, J.W., Appl. Opt. Vol. 33, p. 4096 (1994).

20. Lumme, K. and Rahola, J., Astrophys. J. Vol. 425, p. 653 (1994).

21. Senior, T.B.A., Sarabandi, K., and Ulaby, F.T., "Measuring and Modeling the Backscattering Cross Section of a Leaf," Radio Science, Vol. 22, No. 6, pp. 1109-1116, (1987).

22. Ruck, G.T., Barrick, D.E., Stuart, W.D., and Krichbaum, C.K., *Radar Cross Section Handbook*,
Vol. 2, Battelle Memorial Institute, Plenum Press, New York, NY (1970).

23. Ashcroft, N.W. and Mermin, N.D., *Solid State Physics*, Saunders College, Philadelphia, PA (1976).

24. Kittel, C., *Introduction to Solid State Physics*, 6th ed., John Wiley & Sons, Inc., New York, NY (1986).

25. Ordal, M.A., Long, L.L., Bell, R.J., Bell, S.E., Bell, R.R., Alexander, Jr., R.W., and Ward, C.A., "Optical Properties of the Metals Al, Co, Cu, Au, Fe, Pb, Ni, Pd, Pt, Ag, Ti, and W in the Infrared and Far Infrared," Appl. Opt. Vol. 22, p. 1099, (1983).

26. Bell, R.J., Ordal, M.A., and Alexander, Jr., R.W., "Equations Linking Different Sets of Optical Properties for Nonmagnetic Materials," App. Opt. Vol. 24, p. 3680 (1985).

27. Ordal, M.A., Bell, R.J., Alexander, Jr., R.W., Long, L.L., and Querry, M.R., "Optical Properties of Fourteen Metals in the Infrared and Far Infrared: Al, Co, Cu, Au, Fe, Pb, Mo, Ni, Pd, Pt, Ag, Ti, V, and W," Appl. Opt. Vol. 24, p. 4493, (1985).

28. Larson, D.C., "Size-Dependent Electrical Conduction in Thin Metal Films and Wires," in *Physics of Thin films: Advances in Research and Development*, pp. 81-149, Academic Press, New York, NY (1971).

29. Nordheim, L., Acta. Sci. Ind. No. 131 (1934).

30. Fuchs, K., Proc. Cambridge Phil. Soc. Vol. 34, p. 100 (1938).

31. Zhang, X.-G. and Butler, W.H., "Conductivity of Metallic Films and Multilayers", Phys. Rev. Vol. B51, pp. 10085-10103 (1995).

32. Zhang, X.-G, ORNL, Private Communication (27 Feb 2001).

SOURCE CODE

Disk and Rod Scattering Codes

```
Program Darpadisk1
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
     3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
    Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
     k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
       h0) 10^4);
(*lambmicron = 10^ss;*)
mhopercm1 = 10^xx ;
h = 3 10^-7;
H = 3 10^-5;
h0 = 0;
g1 = Plot3D[
  If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
  PlotLabel -> "Metal Disk h=3nm and D=300nm",
  AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
  PlotPoints -> 20];
h = 10^-6;
H = 10^-4;
g1 = Plot3D[
  If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
```

```
   PlotLabel -> "Metal Disk h=10nm and D=1\[Mu]",
   AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha] "}, PlotPoints -> 20];
h = 10^-6;
H = 2 10^-4;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=10nm and D=2\[Mu]",
   AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha] "}, PlotPoints -> 20];
```

Program Darpadisk2

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
       k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
          h0) 10^4);
(*lambmicron = 10^ss;*)
mhopercm1 = 10^xx ;
h = 10 10^-7;
H = 5 10^-4;
h0 = 0;
g1 = Plot3D[
```

```
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=10nm and D=5\[Mu]",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 H = 3 10^-5;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=10nm and D=300nm",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 H = 3 10^-5;
h = 3 10^-7;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=3nm and D=300nm",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 H = 3 10^-5;
h = 10^-7;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=1nm and D=300nm",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 H = 10^-5;
h = 3 10^-7;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=3nm and D=100nm",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 H = 10^-5;
h = 10^-7;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=1nm and D=100nm",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 h = 3 10^-7;
H = 10^-4;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=3nm and D=1\[Mu]",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 h = 3 10^-7;
H = 4 10^-4;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Disk h=3nm and D=4\[Mu]",
   AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
   PlotPoints -> 20];
 h = 10 10^-7;
H = 4 10^-4;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
```

```
  PlotLabel -> "Metal Disk h=10nm and D=4\[Mu]",
  AxesLabel -> {"\[Lambda](\[Mu])", "log \[Sigma]", "\[Alpha] "},
  PlotPoints -> 20]
```

```
Program Darpadisk3
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
h = 10^xx;
mhopercm1 = 6 10^5;
H = 4 10^-4;
h0 = 0;
g1 = Plot3D[
  If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
  PlotLabel -> "Metal Disk \[Sigma]=6x10^5 and D=4\[Mu]",
  AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5;
```

```
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^5 and D=4\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^4;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^4 and D=4\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^3;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^3 and D=4\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 6 10^5;
H = 10^-4;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=6x10^5 and D=1\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^5;
H = 10^-4;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^5 and D=1\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^4;
H = 10^-4;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^4 and D=1\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^3;
H = 10^-4;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^3 and D=1\[Mu]",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20]
```

```
Program Darpadisk4
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
```

```
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
h = 10^xx;
mhopercm1 = 6 10^5;
H = 3 10^-5;
h0 = 0;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=6x10^5 and D=300nm",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^5 and D=300nm",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^4;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^4 and D=300nm",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=10^3 and D=300nm",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 6 10^5;
H = 1.00001 10^-5;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Disk \[Sigma]=6x10^5 and D=100nm",
   AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5;
g1 = Plot3D[
```

```
  If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
  PlotLabel -> "Metal Disk \[Sigma]=10^5 and D=100nm",
  AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^4;
g1 = Plot3D[
  If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
  PlotLabel -> "Metal Disk \[Sigma]=10^4 and D=100nm",
  AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20];
 mhopercm1 = 10^3;
g1 = Plot3D[
  If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
  PlotLabel -> "Metal Disk \[Sigma]=10^3 and D=100nm",
  AxesLabel -> {"\[Lambda]", "log h", "\[Alpha] "}, PlotPoints -> 20]
```

```
Program Darpadisk5
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
     3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
    Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
```

```
h = 10^-7;
mhopercm1 = 6 10^5;
H = 1.00001 10^xx;
h0 = 0;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
   PlotLabel -> "Metal Disk \[Sigma]=6x10^5 and h=1nm",
   AxesLabel -> {"\[Lambda]", "log D", "\[Alpha] "}, PlotPoints -> 20]
```

Program Darpadisk6

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
     3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
    Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
h = 10 10^-7;
mhopercm1 = 6 10^5;
H = 1.00001 10^xx;
h0 = 0;
g1 = Plot3D[
```

```
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
   PlotLabel -> "Metal Disk \[Sigma]=6x10^5 and h=10nm",
   AxesLabel -> {"\[Lambda]", "log D", "\[Alpha] "}, PlotPoints -> 20];
h = 10^-7;
mhopercm1 = 10^3;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
   PlotLabel -> "Metal Disk \[Sigma]=10^3 and h=1nm",
   AxesLabel -> {"\[Lambda]", "log D", "\[Alpha] "}, PlotPoints -> 20];
Show[%, PlotRange -> {0, 50}];
Show[%, PlotRange -> {0, 15}];
Show[%, PlotRange -> {0, 20}];
h = 10 10^-7;
mhopercm1 = 10^3;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
   PlotLabel -> "Metal Disk \[Sigma]=10^3 and h=10nm",
   AxesLabel -> {"\[Lambda]", "log D", "\[Alpha] "}, PlotPoints -> 20];
Show[%, PlotRange -> {0, 20}];
h = 10 10^-7;
mhopercm1 = 10^5;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
   PlotLabel -> "Metal Disk \[Sigma]=10^5 and h=10nm",
   AxesLabel -> {"\[Lambda]", "log D", "\[Alpha] "}, PlotPoints -> 20];
h = 10 10^-7;
mhopercm1 = 10^4;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
   PlotLabel -> "Metal Disk \[Sigma]=10^4 and h=10nm",
   AxesLabel -> {"\[Lambda]", "log D", "\[Alpha] "}, PlotPoints -> 20];
Show[%, PlotRange -> {0, 50}]
```

Program Darpadisk7

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
   mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
```

```
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
mhopercm1 = 10^xx ;
lambmicron = 14;
h = 10^yy;
H = 4 10^-4;
h0 = 0;
gl = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=4\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 10^-4;
gl = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=1\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 3 10^-5;
gl = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=300nm",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 10^-5;
gl = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
  PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=100nm",
  AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20]
```

Program Darpadisk8

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
```

```
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
mhopercm1 = 10^xx ;
lambmicron = 14;
h = 10^yy;
H = 7 10^-4;
h0 = 0;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
    PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=7\[Mu]",
    AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
g1 = Plot3D[alpha1, {yy, -8, -5}, {xx, 3, 6},
    PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=7\[Mu]",
    AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
h0 = 20 10^-7;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Coated Disk \[Lambda]=14\[Mu],h0=20nm,D=7\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20]
```

Program Darpadisk9

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
```

```
mhopercm =
   mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
    Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
        h0) 10^4);
(*lambmicron = 10^ss;*)
mhopercm1 = 10^xx ;
lambmicron = 14;
h = 10^yy;
H = 5 10^-4;
h0 = 0;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=14\[Mu] and D=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
lambmicron = 7;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=7\[Mu] and D=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
lambmicron = 5;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=5\[Mu] and D=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
lambmicron = 3;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Disk \[Lambda]=3\[Mu] and D=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];


Program Darpadisk10
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
```

```
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
  mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
      3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
       k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
         h0) 10^4);
(*lambmicron = 10^ss;*)
mhopercm1 = 10^xx ;
lambmicron = 14;
h = 10^yy;
H = 7 10^-4;
h0 = 10 10^-7;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
  PlotLabel -> "Coated Disk \[Lambda]=14\[Mu],h0=10nm,D=7\[Mu]",
  AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20]
```

Program Darparod1a

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
```

```
mfp = vf*tau1;
kap = h/mfp;
mhopercm = mhopercm1/(1 + mfp/h);
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
          BesselJ[-1,
           x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
             I BesselY[0, x]) - (BesselJ[-1, x] +
            I BesselY[-1, x])) +
      2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
            BesselJ[j - 1,
             x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
                I BesselY[j, x]) - (BesselJ[j - 1, x] +
              I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
          BesselJ[-1,
           x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
             I BesselY[0, x]) - (BesselJ[-1, x] +
            I BesselY[-1, x])) +
      2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) +
                j/x)BesselJ[j, x] -
            BesselJ[j - 1,
             x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
               j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
               j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
qe = (qe1 + qe2)/3;
alpha1 = qe/(Pi h 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/
        4) H h^2 10^4);
h = 30 10^-7;
H = 4 10^-4;
mhopercm1 = 10^xx ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=4\[Mu] and h=30nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha] "},
    PlotPoints -> 20];
h = 10 10^-7;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=4\[Mu] and h=10nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
```

```
h = 10 10^-7;
H = 10^-4;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=1\[Mu] and h=10nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 30 10^-7;
H = 10^-4;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=1\[Mu] and h=30nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 30 10^-7;
H = 3 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=300nm and h=30nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 10 10^-7;
H = 3 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=300nm and h=10nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 10^-7;
H = 3 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=300nm and h=1nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 3 10^-7;
H = 3 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=300nm and h=3nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 10^-7;
H = 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
    PlotLabel -> "Metal Rod H=100nm and h=1nm",
    AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
    PlotPoints -> 20];
h = 3 10^-7;
H = 10^-5;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, 3, 6},
   PlotLabel -> "Metal Rod H=100nm and h=3nm",
   AxesLabel -> {"\[Lambda]", "log \[Sigma]", "\[Alpha]  "},
   PlotPoints -> 20]
```

Program Darparod2a

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1/(1 + mfp/h);
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
          BesselJ[-1,
           x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
              I BesselY[0, x]) - (BesselJ[-1, x] +
            I BesselY[-1, x])) +
      2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
            BesselJ[j - 1,
              x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
                  I BesselY[j, x]) - (BesselJ[j - 1, x] +
               I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
          BesselJ[-1,
           x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
              I BesselY[0, x]) - (BesselJ[-1, x] +
            I BesselY[-1, x])) +
      2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) +
                 j/x)BesselJ[j, x] -
            BesselJ[j - 1,
              x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
                j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
                j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
qe = (qe1 + qe2)/3;
alpha1 = qe/(Pi (h + h0) 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/
        4) H (h + h0)^2 10^4);
h = 10^xx;
mhopercm1 = 6 10^5 ;
H = 4 10^-4;
h0 = 0;
```

```
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=6x10^5,H=4\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5 ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^5,H=4\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^4 ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^4,H=4\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3 ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^3,H=4\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 6 10^5 ;
H = 10^-4;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=6x10^5,H=1\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5 ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^5,H=1\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^4 ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^4,H=1\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3 ;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^3,H=1\[Mu]",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 6 10^5;
H = 3 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=6x10^5,H=300nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^5,H=300nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^4;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^4,H=300nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
```

```
mhopercm1 = 10^3;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^3,H=300nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 6 10^5;
H = 1.00001 10^-5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=6x10^5,H=100nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^5,H=100nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^4;
g1 = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
     PlotLabel -> "Metal Rod \[Sigma]=10^4,H=100nm",
    AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -5, -8},
   PlotLabel -> "Metal Rod \[Sigma]=10^3,H=100nm",
   AxesLabel -> {"\[Lambda]", "Log h", "\[Alpha] "}, PlotPoints -> 20]
```

Program Darparod3a

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1/(1 + mfp/h);
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
            BesselJ[-1,
             x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
                I BesselY[0, x]) - (BesselJ[-1, x] +
              I BesselY[-1, x])) +
        2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
              BesselJ[j - 1,
                x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
```

```
          I BesselY[j, x]) - (BesselJ[j - 1, x] +
        I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
        BesselJ[-1,
         x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
          I BesselY[0, x]) - (BesselJ[-1, x] +
         I BesselY[-1, x])) +
    2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) +
           j/x)BesselJ[j, x] -
         BesselJ[j - 1,
          x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
           j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
           j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
qe = (qe1 + qe2)/3;
alpha1 = qe/(Pi (h + h0) 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
    k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/
      4) H (h + h0)^2 10^4);
h = 10^-7;
mhopercm1 = 6 10^5 ;
H = 1.00001 10^xx;
h0 = 0;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
    PlotLabel -> "Metal Rod \[Sigma]=6x10^5,h=1nm",
   AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 30];
mhopercm1 = 10^5 ;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
    PlotLabel -> "Metal Rod \[Sigma]=10^5,h=1nm",
   AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 30];
mhopercm1 = 10^4 ;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
    PlotLabel -> "Metal Rod \[Sigma]=10^4,h=1nm",
   AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3 ;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
    PlotLabel -> "Metal Rod \[Sigma]=10^3,h=1nm",
   AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 6 10^5 ;
h = 10 10^-7;
g1 = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
    PlotLabel -> "Metal Rod \[Sigma]=6x10^5,h=10nm",
   AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 40];
mhopercm1 = 10^5 ;
h = 10 10^-7;
```

```
gl = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
     PlotLabel -> "Metal Rod \[Sigma]=10^5,h=10nm",
    AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 30];
mhopercm1 = 10^4 ;
h = 10 10^-7;
gl = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
     PlotLabel -> "Metal Rod \[Sigma]=10^4,h=10nm",
    AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3 ;
h = 10 10^-7;
gl = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -6},
     PlotLabel -> "Metal Rod \[Sigma]=10^3,h=10nm",
    AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 6 10^5 ;
h = 30 10^-7;
gl = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1,
      14}, {xx, -3, -5.5}, PlotLabel -> "Metal Rod \[Sigma]=6x10^5,h=30nm",
    AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^5 ;
h = 30 10^-7;
gl = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1,
      14}, {xx, -3, -5.5}, PlotLabel -> "Metal Rod \[Sigma]=10^5,h=30nm",
    AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^4 ;
h = 30 10^-7;
gl = Plot3D[
    If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1,
      14}, {xx, -3, -5.5}, PlotLabel -> "Metal Rod \[Sigma]=10^4,h=30nm",
    AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20];
mhopercm1 = 10^3 ;
h = 30 10^-7;
gl = Plot3D[
   If[alpha1 <= alpha2, alpha1, alpha2], {lambmicron, 1, 14}, {xx, -3, -5.5},
    PlotLabel -> "Metal Rod \[Sigma]=10^3,h=30nm",
   AxesLabel -> {"\[Lambda]", "Log H", "\[Alpha] "}, PlotPoints -> 20]
```

```
Program Darparod4a
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1/(1 + mfp/h);
```

```
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
           BesselJ[-1,
            x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
               I BesselY[0, x]) - (BesselJ[-1, x] +
             I BesselY[-1, x])) +
       2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
             BesselJ[j - 1,
              x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
                 I BesselY[j, x]) - (BesselJ[j - 1, x] +
               I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
           BesselJ[-1,
            x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
               I BesselY[0, x]) - (BesselJ[-1, x] +
             I BesselY[-1, x])) +
       2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) +
                 j/x)BesselJ[j, x] -
             BesselJ[j - 1,
              x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
                 j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
                 j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
qe = (qe1 + qe2)/3;
alpha1 = qe/(Pi (h + h0) 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
       k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/
          4) H (h + h0)^2 10^4);
mhopercm1 = 10^xx ;
lambmicron = 14;
h = 10^yy;
h0 = 0;
H = 20 10^-4;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
    PlotLabel -> "Metal Rod \[Lambda]=14\[Mu] and H=20\[Mu]",
    AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 4 10^-4;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
    PlotLabel -> "Metal Rod \[Lambda]=14\[Mu] and H=4\[Mu]",
    AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 10^-4;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
    PlotLabel -> "Metal Rod \[Lambda]=14\[Mu] and H=1\[Mu]",
    AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 3 10^-5;
```

```
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
    PlotLabel -> "Metal Rod \[Lambda]=14\[Mu] and H=300nm",
    AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
H = 1.00001 10^-5;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Rod \[Lambda]=14\[Mu] and H=100nm",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20]
```

```
Program Darparod6a
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1/(1 + mfp/h);
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
         BesselJ[-1,
          x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
            I BesselY[0, x]) - (BesselJ[-1, x] + I BesselY[-1, x])) +
      2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
            BesselJ[j - 1,
             x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
               I BesselY[j, x]) - (BesselJ[j - 1, x] +
              I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
         BesselJ[-1,
          x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
            I BesselY[0, x]) - (BesselJ[-1, x] + I BesselY[-1, x])) +
      2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) + j/x)BesselJ[
              j, x] -
            BesselJ[j - 1,
             x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
               j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
               j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
qe = (qe1 + qe2)/3;
alpha1 = qe/(Pi (h + h0^2/h) 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
```

```
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
     k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H (h^2 +
       h0^2) 10^4);
mhopercm1 = 10^xx ;
lambmicron = 3;
h = 10^yy;
h0 = 0;
H = 5 10^-4;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Rod \[Lambda]=3\[Mu],H=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
lambmicron = 5;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Rod \[Lambda]=5\[Mu],H=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
lambmicron = 7;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Rod \[Lambda]=7\[Mu],H=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
lambmicron = 14;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, 3, 6},
   PlotLabel -> "Metal Rod \[Lambda]=14\[Mu],H=5\[Mu]",
   AxesLabel -> {"log h", "log \[Sigma]", "\[Alpha]"}, PlotPoints -> 20];
```

```
Program DiskhvH
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
kap = h/mfp;
mhopercm =
   mhopercm1(1 - .75(kap - kap^3/12)ExpIntegralEi[-kap] -
       3(1 - Exp[-kap])/(8kap) - (5/8 + kap/16 - kap^2/16)Exp[-kap]) ;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
```

```
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
       k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
          h0) 10^4);
(*lambmicron = 10^ss;*)
h = 10^yy;
H = 1.00001 10^xx;
h0 = 0;
mhopercm1 = 10^5 ;
lambmicron = 3;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Disk \[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Disk \[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(\[Lambda]<=3H transition)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(\[Lambda]<=3H transition)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
     AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
g1 = Plot3D[1/(1/alpha1 + 1/alpha2), {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(1/\[Alpha]1+1/\[Alpha]2 \
transition)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[1/(1/alpha1 + 1/alpha2), {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(1/\[Alpha]1+1/\[Alpha]2 \
transition)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
```

```
Program DiskhvH2
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
```

```
A = H/h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
a2 = (2Pi h eps^.5/lamb)(1 - Sin[th]^2/eps)^.5;
z1 = (1/Cos[th] + Cos[th])/2;
z2 = (1/(1 - Sin[th]^2/eps)^.5 + (1 - Sin[th]^2/eps)^.5)/eps^.5;
rr = I(z1^2 - z2^2)Tan[a2]/(2z1 z2 - I(z1^2 + z2^2)Tan[a2]);
tt = 2z1 z2/(2z1 z2 Cos[a2] - I(z1^2 + z2^2)Sin[a2]);
R = Abs[rr]^2;
T = Abs[tt]^2;
alpha1 = NIntegrate[
     Cos[th](1 - T)10^-4/(h + h0), {th, 10^-4, Pi/2 - 10^-4}]/(Pi/2);
ee = (1 - 1/A^2)^.5;
ge = (1/ee^2 - 1)^.5;
L1 = ge/(2ee^2)(Pi/2 - ArcTan[ge]) - ge^2/2;
L2 = L1;
L3 = 1 - L1 - L2;
p1 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H^2 (h + h0)(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/4) H^2(h +
         h0) 10^4);
(*lambmicron = 10^ss;*)
h = 10^yy;
H = 1.00001 10^xx;
h0 = 0;
mhopercm1 = 10^5 ;
lambmicron = 3;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(No size effects)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(No size effects)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(No size effects)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Disk(No size effects)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
```

```
Program RodhvH2
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1/(1 + mfp/h);
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
        BesselJ[-1,
         x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
           I BesselY[0, x]) - (BesselJ[-1, x] +
          I BesselY[-1, x])) +
     2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
          BesselJ[j - 1,
           x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
              I BesselY[j, x]) - (BesselJ[j - 1, x] +
            I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
        BesselJ[-1,
         x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
           I BesselY[0, x]) - (BesselJ[-1, x] +
          I BesselY[-1, x])) +
     2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) +
              j/x)BesselJ[j, x] -
          BesselJ[j - 1,
           x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
             j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
             j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
(*combine results for random orientation in plane with G =
  S/4 = 2A/4 = A/2 for 3D random orientation*)
qe = (qe1 + qe2)/4;
alpha1 = qe/(Pi (h + h0) 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
      k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/
        4) H (h + h0)^2 10^4);
h = 10^yy;
H = 1.00001 10^xx;
```

```
h0 = 0;
mhopercm1 = 10^5 ;
lambmicron = 3;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Rod \[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Rod \[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Rod(\[Lambda]<=3H transition)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Rod(\[Lambda]<=3H transition)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
g1 = Plot3D[1/(1/alpha1 + 1/alpha2), {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Rod(1/\[Alpha]1+1/\[Alpha]2 \
transition)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[1/(1/alpha1 + 1/alpha2), {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Rod(1/\[Alpha]1+1/\[Alpha]2 \
transition)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
```

Program RodhvH3

```
c = 3 10^10;
lamb = lambmicron/10^4;
omeg = 2 Pi c/lamb;
k = 2 Pi / lamb;
rsa0 = 2.5;
vf = 4.2 10^8/rsa0;
A = H/h;
x = k h;
tau1 = .22*mhopercm1*10^-6*rsa0^3*10^-14;
mfp = vf*tau1;
mhopercm = mhopercm1;
tau = mhopercm tau1/mhopercm1;
sig0 = 10^-9 c^2 mhopercm;
sig = sig0/ (1 - I omeg tau);
eps = 1 + I 4 Pi sig/omeg;
m = eps^0.5;
z = m x;
qe1 = (2/x)Re[((m BesselJ[-1, z]/BesselJ[0, z]) BesselJ[0, x] -
        BesselJ[-1,
```

```
            x])/((m BesselJ[-1, z]/BesselJ[0, z])(BesselJ[0, x] +
              I BesselY[0, x]) - (BesselJ[-1, x] +
             I BesselY[-1, x])) +
       2Sum[((m BesselJ[j - 1, z]/BesselJ[j, z])BesselJ[j, x] -
             BesselJ[j - 1,
              x])/((m BesselJ[j - 1, z]/BesselJ[j, z])(BesselJ[j, x] +
                 I BesselY[j, x]) - (BesselJ[j - 1, x] +
               I BesselY[j - 1, x])), {j, 1, 19}]];
qe2 = (2/x)Re[(BesselJ[-1, z] BesselJ[0, x]/(m BesselJ[0, z]) -
          BesselJ[-1,
            x])/( (BesselJ[-1, z]/(m BesselJ[0, z]))(BesselJ[0, x] +
              I BesselY[0, x]) - (BesselJ[-1, x] +
            I BesselY[-1, x])) +
       2Sum[(( BesselJ[j - 1, z]/(m BesselJ[j, z]) - j /(m z) +
                j/x)BesselJ[j, x] -
            BesselJ[j - 1,
              x])/( (BesselJ[j - 1, z]/(m BesselJ[j, z]) - j/(m z) +
                j/x)(BesselJ[j, x] + I BesselY[j, x]) - (BesselJ[
                j - 1, x] + I BesselY[j - 1, x])), {j, 1, 19}]];
(*combine results for random orientation in plane with G =
   S/4 = 2A/4 = A/2 for 3D random orientation*)
qe = (qe1 + qe2)/4;
alpha1 = qe/(Pi (h + h0) 10^4/4);
ee = (1 - 1/A^2)^.5;
L1 = (1/ee^2 - 1)(-1 + Log[(1 + ee)/(1 - ee)]/(2ee));
L2 = (1 - L1)/2;
L3 = L2;
p1 = (Pi/2) H h^2(eps - 1)/(3 + 3L1(eps - 1));
p2 = (Pi/2) H h^2(eps - 1)/(3 + 3L2(eps - 1));
p3 = (Pi/2) H h^2(eps - 1)/(3 + 3L3(eps - 1));
alpha2 = (k Im[p1 + p2 + p3]/3 +
        k^4(Abs[p1]^2 + Abs[p2]^2 + Abs[p3]^2)/(18 Pi))/((Pi/
           4) H (h + h0)^2 10^4);
h = 10^yy;
H = 1.00001 10^xx;
h0 = 0;
mhopercm1 = 10^5 ;
lambmicron = 3;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Rod(No size effects)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
     AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[alpha1 <= alpha2, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Rod(No size effects)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Rod(No size effects)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
     AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[If[lamb <= 3H, alpha1, alpha2], {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
     "Metal Rod(No size effects)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 3;
```

```
g1 = Plot3D[1/(1/alpha1 + 1/alpha2), {yy, -8, -5}, {xx, -1, -5},
    PlotLabel -> "Metal Rod(No size effects)\[Sigma]=10^5,\[Lambda]=3\[Mu]",
     AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
lambmicron = 14;
g1 = Plot3D[1/(1/alpha1 + 1/alpha2), {yy, -8, -5}, {xx, -1, -5},
    PlotLabel ->
      "Metal Rod(No size effects)\[Sigma]=10^5,\[Lambda]=14\[Mu]",
    AxesLabel -> {"Log h", "Log H", "\[Alpha] "}, PlotPoints -> 30];
```